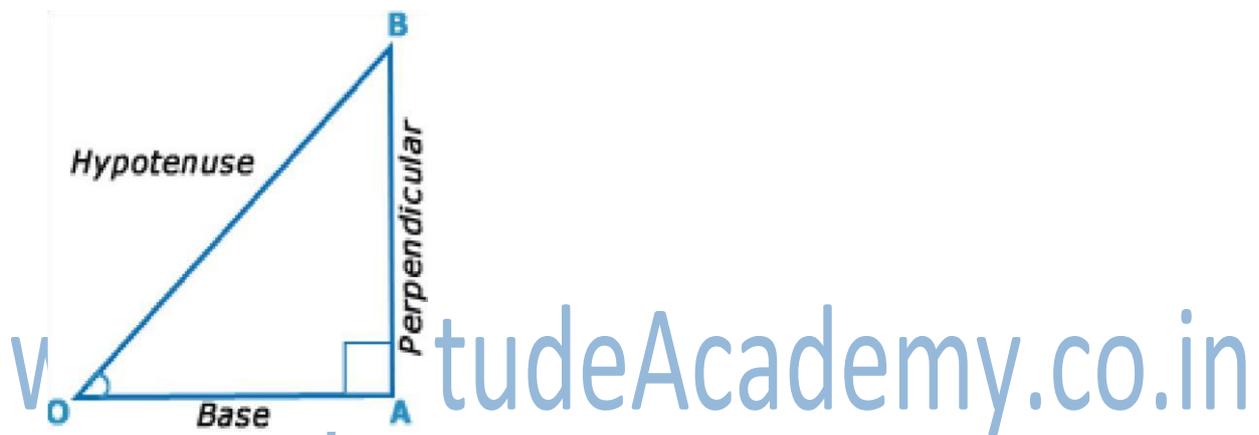


Module Name: HEIGHTS AND DISTANCE

PREREQUISITES:

1. Trigonometry:

In a right angled ΔOAB , where $\angle BOA = \theta$,



i . $\sin \theta = \text{Perpendicular} / \text{Hypotenuse} = AB/OB$ ii

. $\cos \theta = \text{Base} / \text{Hypotenuse} = OA/OB$ iii . $\tan \theta =$

$\text{Perpendicular} / \text{Base} = AB/OA$ iv . $\text{cosec } \theta =$

$1/\sin \theta = OB/AB$ v . $\sec \theta = 1/\cos \theta =$

OB/OA vi . $\cot \theta = 1/\tan \theta = OA/AB$

Reasoning and Quantitative aptitude Heights and Distance

Trigonometrical Identities:

i . $\sin^2 \theta + \cos^2 \theta = 1$ ii .

$1 + \tan^2 \theta = \sec^2 \theta$ iii . $1 +$

$\cot^2 \theta = \operatorname{cosec}^2 \theta$

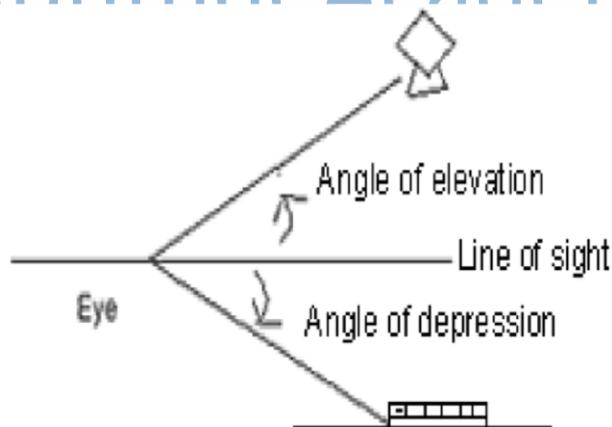
Height and Distance:

Some times we are required to find the height of a tower, tree, building and distance of a ship from light house, width of a river etc. We cannot measure them accurately, though we can find them using the knowledge of trigonometric ratio.

Line of sight:

When we see an object standing on the ground. The line of sight is the line from our eye to the object, we see.

Angle of Elevation:



When the object is above the horizontal level of our eye, we have to turn our head upwards to see an object. In this process, our eyes move through an angle which is called angle of elevation.

Angle of Depression:

When the object is on the ground and the observer is on a building then the object is below the level of the eye of the observer. The observer has to turn his head downward to see the object. In doing so, his eyes move through an angle which is called angle of depression.

Reasoning and Quantitative aptitude Heights and Distance

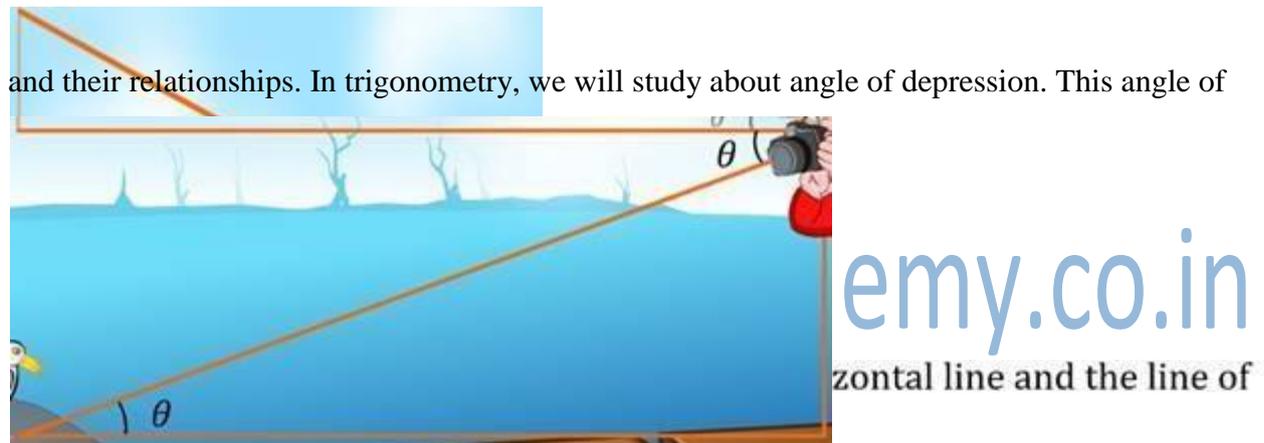
Heights and Distance

Trigonometry is the study of **relationships between the sides and angles** of a triangle. It is used in **geography** and in **navigation**. It is also used in **constructing maps, determine the position of an Island** in relation to the **longitudes and latitudes**.

Here **trigonometry** is used for finding the **heights and distances** of various objects without actually measuring it.

The **line joining the observer's eye and the object observed** is known as **Line of Sight**. The **angle between the horizontal line and the line of sight** which is above the **observer's eye** is known as **Angle of Elevation**.

The **angle between the horizontal line and the line of sight** which is below the **observer's eye** is called **Angle of Depression**.



Angle of Depression:

Angle of depression: The angle between the horizontal line and line of sight which is below the observer's eye.

depression is very helpful for related real life problems. Let us see about angle of depression

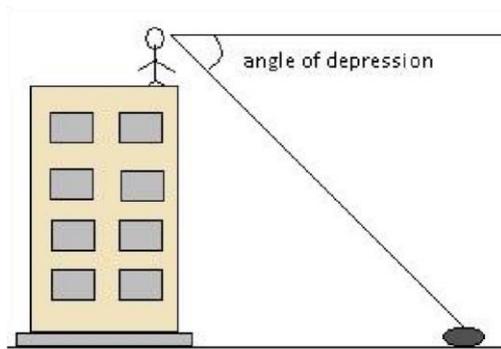
Trigonometry is an important study in math. Trigonometry is study about angles in detail.

Angle of depression is a term studied mainly in trigonometry where “**depression**” means “**fall**” or “**drop**”. Angle of depression is an angle between horizontal line and view the object from the horizontal line. Angle of depression is mainly used for obtaining the distance of the two objects where we only know their angle and an object's distance from the ground.

Let us see some of the examples problems of angle of depression in trigonometry.

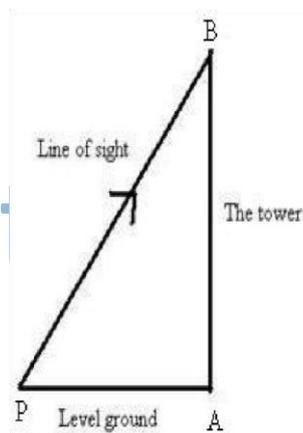
See the diagram below :

Reasoning and Quantitative aptitude Heights and Distance



Line of Sight:

The **line of sight** is a straight line along which an observer observes an object . Suppose an observer observes an object on a tower. The line joining the eyes of the observer and the object is called the line of sight. In the following figure AB represents a tower and P be the position of a man who is standing on the horizontal ground watching a bird at the top of the tower.



Here the line joining the eyes of the man and the bird is called the line of sight. So PB represents the line of sight.

Angle of elevation:

The object may be above or below the horizontal. If the object is above the horizontal, the angle between the line of sight and the horizontal is called the angle of elevation.

Reasoning and Quantitative aptitude Heights and Distance

Terms used in height and distance:

Horizontal ray:

A ray parallel to the surface of the earth emerging from the eye of the observer is known as horizontal ray.

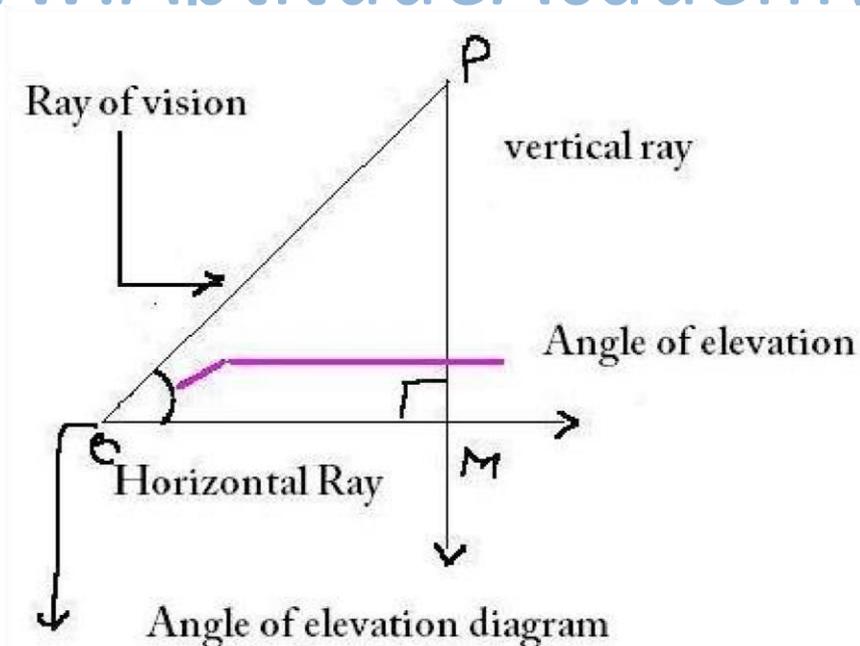
Ray of vision:

The ray from the eye of the observer towards the object under observation is known as the ray of vision or ray of sight.

Angle of Elevation:

If the object under observation is above the horizontal ray passing through the point of observation, the measure of the angle formed by the horizontal ray and the ray of vision is known as angle of

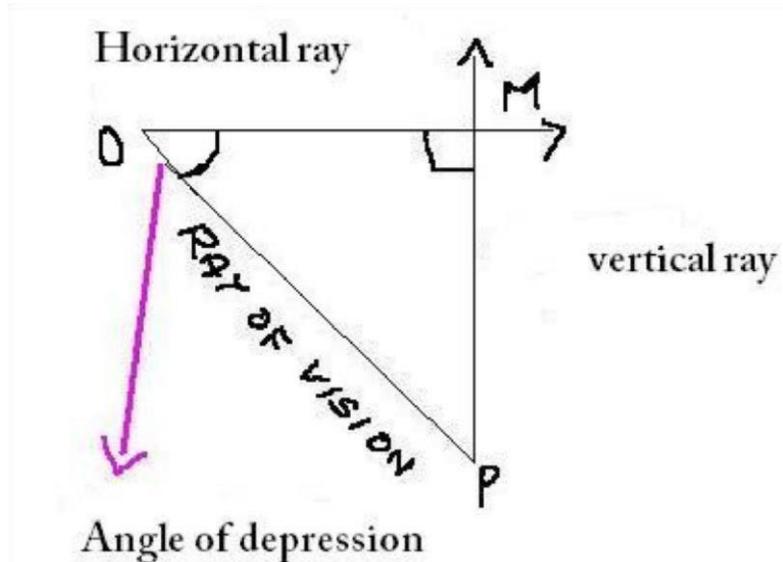
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elevation..

Reasoning and Quantitative aptitude Heights and Distance

Angle of depression: If the object under observation is below the horizontal ray passing through the point of observation, the measure of the angle formed by the horizontal ray and the ray of vision is known as angle of depression.



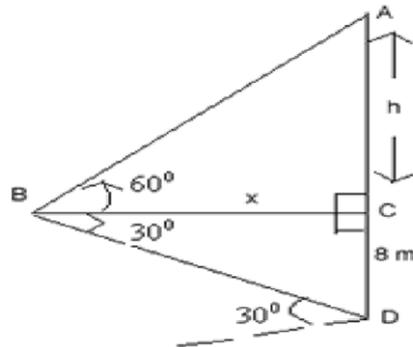
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Example . A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill.

Solution: - Let B be man, D the base of the hill, x be the distance of hill from the ship and

Reasoning and Quantitative aptitude Heights and Distance

h



+ 8 be the height of the hill.

$$\angle ABC = 60^\circ, \angle DBC = 30^\circ$$

In $\triangle ACB$

$$\tan 60^\circ = AC/BC$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \text{ ----- (i)}$$

In $\triangle BCD$

$$\tan 30^\circ = CD/BC$$

$$\frac{1}{\sqrt{3}} = \frac{8}{x} \Rightarrow x = 8\sqrt{3}$$

$$\text{from (i) } h = x\sqrt{3}$$

$$= (8\sqrt{3})\sqrt{3}$$

$$= 24$$

∴

$$\begin{aligned} \text{Height of the hill} &= h + 8 = \\ &= 24 + 8 = 32\text{m} \end{aligned}$$

$$\text{Distance of the hill from the ship} = 8\sqrt{3}\text{m}$$

Reasoning and Quantitative aptitude Heights and Distance

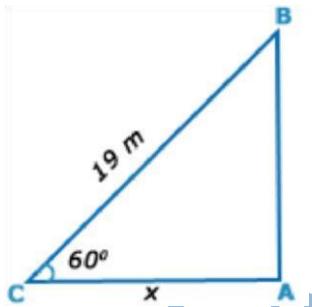
SOLVED PROBLEMS ON HEIGHTS AND DISTANCE:

1. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall. (WIPRO-2009)

A. 9 m B. 9.5 m

C. 10.5 m D. 12 m

Solution



Let AB be the wall and BC be the ladder.

Then, $\angle ABC = 60^\circ$
and, BC = 19 m.;

AC = x metres

$$= \cos 60^\circ$$

$$= x / 19$$

$$AC/BC = 1 / 2$$

$$x = 19/2$$

$$= 9.5 \text{ m.}$$

The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 2.

4.6 m away from the wall. The length of the ladder is ? (CTS 2007)

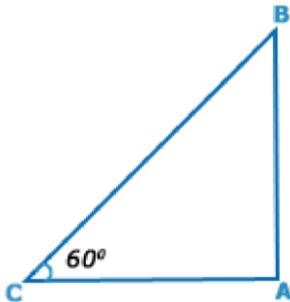
A. 2.3 m B. 4.6 m

Reasoning and Quantitative aptitude Heights and Distance

C. 7.8 m D. 9.2 m

Solution

Let AB be the wall and BC be the ladder.



$$\begin{aligned}\text{Then, } \angle ABC &= 60^\circ \\ \frac{AC}{BC} &= \cos 60^\circ \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{AC}{BC} &= \frac{BC}{2 \times AC} \\ &= \frac{2 \times 4.6}{9.2}\end{aligned}$$

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=9.2 m The angle of elevation of the sun, when the length of the

shadow of a tree is $\sqrt{3}$ times the

3. height of the tree is .(KEANE 2003)

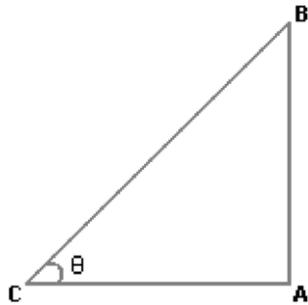
A. 30° B. 45°

C. 60° D. 90°

Solution

Let AB be the tree and AC be its shadow.

Reasoning and Quantitative aptitude Heights and Distance



Then, $\angle ABC = \theta$.
 $= \sqrt{3}$

Then, $AC/AB \cot \theta = \sqrt{3}$
 $\theta = 30^\circ$

If the height of a pole is $2\sqrt{3}$ meters and the length of its shadow is 2 meters, find the angle of elevation of the sun. (CTS 1998)

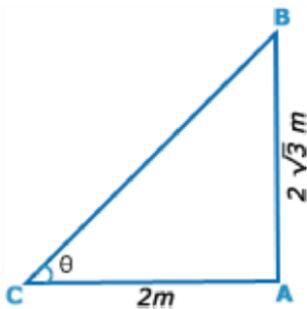
A. 30° B. 45°

C. 60° D. 90°

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Solution

Let AB be the pole and AC be its shadow.



Then, $\angle ACB = \theta$.

Then, $AB = 2\sqrt{3}$ m, $AC = 2$ m, $\tan \theta = AB/AC = 2\sqrt{3}/2$

$= \sqrt{3}$ $\theta = 60^\circ$

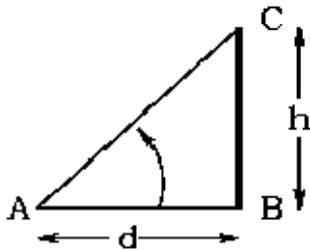
Reasoning and Quantitative aptitude Heights and Distance

One of AB, AD and CD must have been given. So, the data is inadequate.

7. A man is walking along a straight road. He notices the top of a tower subtending an angle $A = 60^\circ$ with the ground at the point where he is standing. If the height of the tower is $h = 30$ m, then what is the distance (in meters) of the man from the tower? (INFOSYSTEMS 2000)

A. 15.23 B. 17.32 C. 20

D. 25.32 Answer: 17.32 Solution:



Let BC represent the tower with height $h = 30$ m, and A represent the point where the man is standing. $AB = d$ denotes the distance of the man from tower. The angle subtended by the tower is $A = 60^\circ$.

From trigonometry,

$$\tan A = \tan 60^\circ = h / d = \sqrt{3}$$

So $d = 30 / \sqrt{3}$ m.

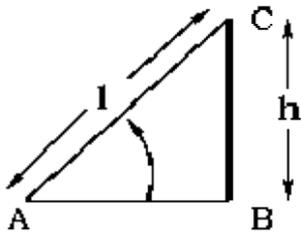
Hence the distance of the man from the tower is 17.32 m.

8. A little boy is flying a kite. The string of the kite makes an angle of 30° with the ground. If the height of the kite is $h = 24$ m, find the length (in meters) of the string that the boy has used. (CTS 1999)

A. 56 B. 28 C. 48 D. 24

Reasoning and Quantitative aptitude Heights and Distance

Answer: 48 Solution:



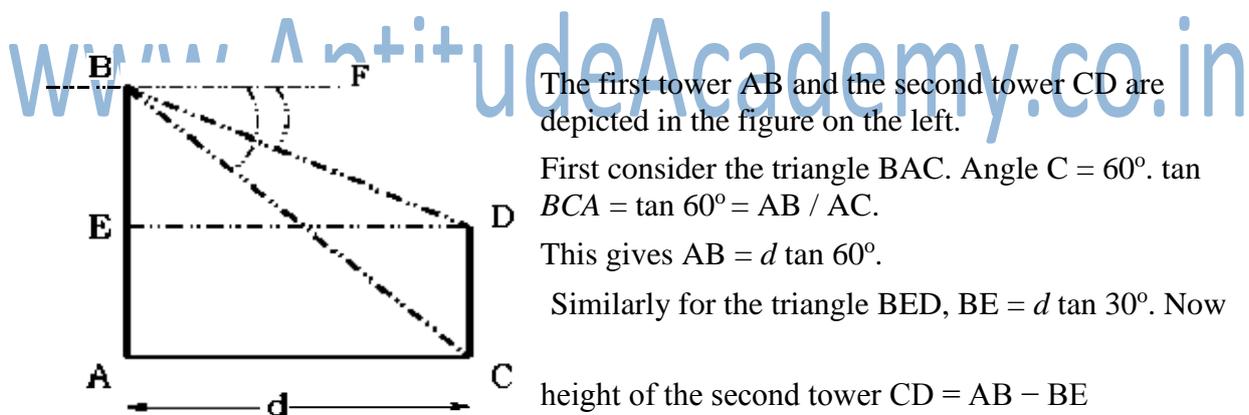
If the kite is at C and the boy is at A, then $AC = l$ represents the length of the string and $BC = h$ represents the height of the kite.

From the figure, $\sin A = \sin 30^\circ = h / l = 1 / 2$. Hence the length of the string used by the little boy is $l = 2h = 2(24) = 48$ m.

9. Two towers face each other separated by a distance $d = 25$ m. As seen from the top of the first tower, the angle of depression of the second tower's base is 60° and that of the top is 30° . What is the height (in meters) of the second tower? (IBM 1998)

A.28.87 B.25.52 C. 25 D. 28

Answer: 28.87 Solution:



The first tower AB and the second tower CD are depicted in the figure on the left.

First consider the triangle BAC. Angle C = 60° . $\tan BCA = \tan 60^\circ = AB / AC$.

This gives $AB = d \tan 60^\circ$.

Similarly for the triangle BED, $BE = d \tan 30^\circ$. Now

height of the second tower $CD = AB - BE$

$$= d (\tan 60^\circ - \tan 30^\circ)$$

$$= 25 (\sqrt{3} - 1/\sqrt{3}) = 25 \times 2/\sqrt{3} = 28.87 \text{ m.}$$

10. A ship of height $h = 12$ m is sighted from a lighthouse. From the top of the lighthouse, the angle of depression to the top of the mast and the base of the ship equal 30° and 45°

respectively. How far is the ship from the lighthouse (in meters)? (MPHASYS 2001) A.38.5

B. 22.52 C. 12.39 D. 28.39

Reasoning and Quantitative aptitude Heights and Distance

Answer: 28.39 Solution:

Let AB represent the lighthouse and CD represent the ship. From the figure, $\tan BCA = \tan 45^\circ = AB / AC$.

Similarly for the triangle BED, $\tan BDE = \tan 30^\circ = BE / ED$.

Now, $AC = ED = d$.

Height of the ship = CD

$$= AB - BE = d (\tan 45^\circ - \tan 30^\circ) = 12 \text{ m.}$$

Thus distance of the ship from the lighthouse $d =$

$$12 / (1 - 1 / \sqrt{3}) = 28.39 \text{ m}$$

11. Two men on opposite sides of a TV tower of height 26 m notice the angle of elevation of the top of this tower to be 45° and 60° respectively. Find the distance (in meters) between the two men.

(ACCENTURE 2002)

- A. 42 B. 41.01 C. 26 D. 45

Answer: 41.01 Solution:

The situation is depicted in the figure with CD representing the tower and AB being the distance between the two men. For triangle ACD, $\tan A = \tan 60^\circ = CD / AD$. Similarly for triangle BCD, $\tan B = \tan 45^\circ = CD / DB$.

The distance between the two men is

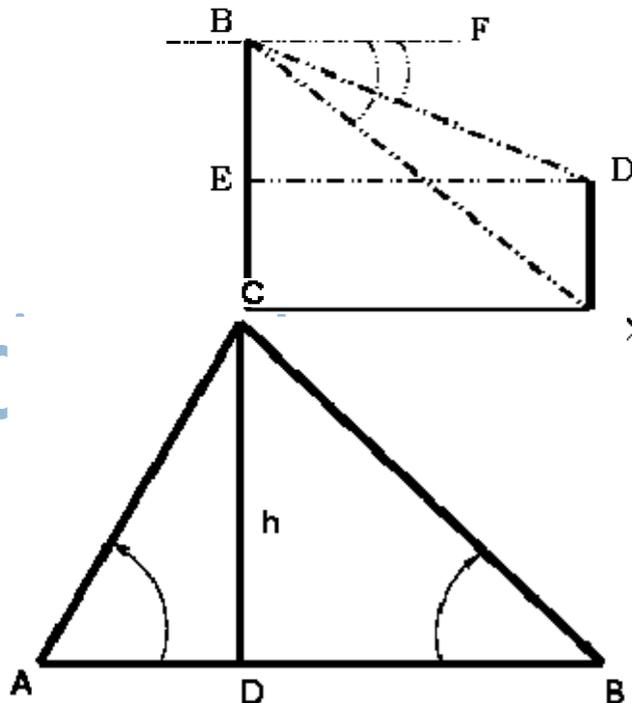
$$AB = AD + DB$$

$$= (CD / \tan 60^\circ) + (CD / \tan 45^\circ) = (26 / \sqrt{3}) + (26 / 1) = 41.01 \text{ m.}$$

12. Two men on the same side of a tall building notice the angle of elevation to the top of the building to be 30° and 60° respectively. If the height of the building is known to be $h = 120$ m, find the distance (in meters) between the two men. **(SASKEEN 2003)**

- A. 138.56 B. 135 C. 120 D. 130.56

Answer: 138.56 Solution:



Reasoning and Quantitative aptitude Heights and Distance

In the figure, A and B represent the two men and CD the tall building.

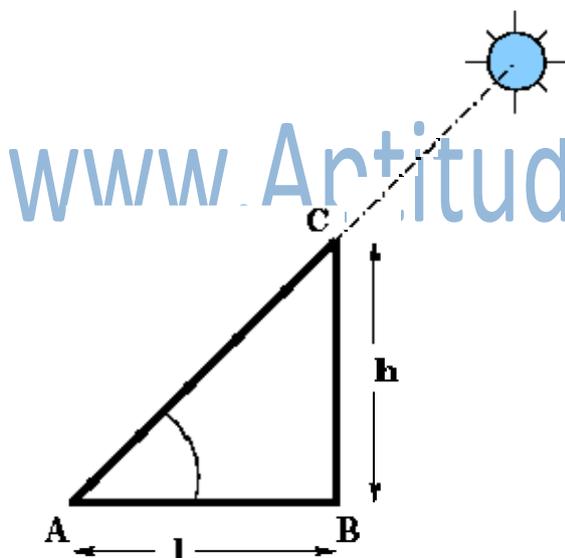
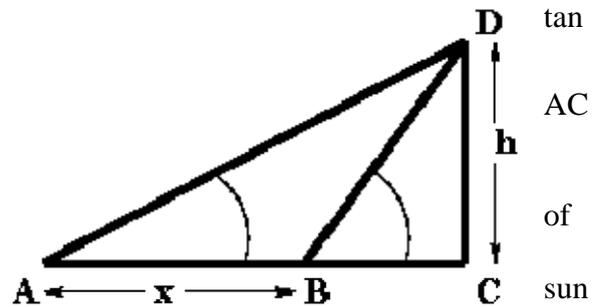
$\tan A = \tan 30^\circ = DC / AC = h / AC$; and $\tan B = 60^\circ = DC / BC = h / BC$.

Now the distance between the men is $AB = x = BC - AC = (h / \tan 30^\circ) - (h / \tan 60^\circ) = (120 \sqrt{3}) - (120 / \sqrt{3}) = 138.56$ m.

13. A pole of height $h = 50$ ft has a shadow length $l = 50.00$ ft at a particular instant of time. Find the angle of elevation (in degrees) of the sun at this point of time. (CSC 2000)

- A. 90 B. 45 C. 60 D.30 **Answer:** 45

Solution:



In the figure, BC represents the pole and AB its shadow. $\tan A = BC / AB$

$= h / l = 50 / 50.00 = 1.000$ From trigonometric tables, we note that $\tan A = 1.000$ for $A = 45^\circ$.

Hence the angle of elevation of the sun at this point of time is 45° .

14. You are stationed at a radar base and you observe an unidentified plane at an altitude $h = 6000$ m flying towards your radar base at an angle of elevation $= 30^\circ$. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation $= 60^\circ$ maintaining the same

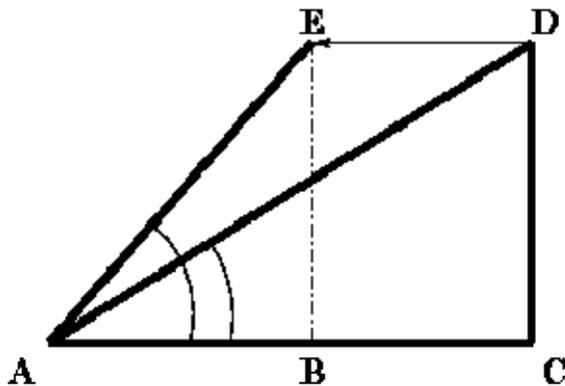
altitude. What is the speed (in m/s) of the plane? (DELL 2002)

- A. 6000 B. 6928 C. 115.47 D. 135.5 **Answer:** 115.47

Solution:

In the figure, the radar base is at point A. The plane is at point D in the first sweep and at point E in the second sweep. The distance it covers in the one minute interval is DE.

Reasoning and Quantitative aptitude Heights and Distance



From the figure, $\tan DAC = \tan 30^\circ = DC / AC = h / AC$.

Similarly, $\tan EAB = \tan 60^\circ = EB / AB = h / AB$. Distance covered by the plane in one minute = $DE = AC - AB = (h / \tan 30^\circ) - (h / \tan 60^\circ)$

$$= (6000 \sqrt{3}) - (6000 / \sqrt{3}) = 6928.20 \text{ m.}$$

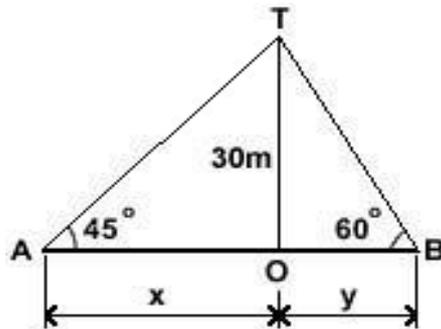
The velocity of the plane is given by $V = \text{distance covered} / \text{time taken} = DE / 60 = 115.47 \text{ m/s.}$

15. The angle of elevation of the top of a tower 30 m high, from two points on the level ground on its opposite sides are 45 degrees and 60 degrees. What is the distance between the two points? (IBM 1999)

- (1) 30
- (2) 51.96
- (3) 47.32
- (4) 81.96

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Correct choice (3). Correct Answer - 47.32 Solution:



Let OT be the tower.

Therefore, Height of tower = $OT = 30 \text{ m}$

Let A and B be the two points on the level ground on the opposite side of tower OT.

Reasoning and Quantitative aptitude Heights and Distance

Then, angle of elevation from A = $\angle TAO = 45^\circ$ and angle of elevation from B = $\angle TBO = 60^\circ$

Distance between AB = AO + OB = x + y (say)

Now, in right triangle ATO,

$$\tan 45^\circ = \frac{OT}{AO} = \frac{30}{x}$$

$$\Rightarrow x = \frac{30}{\tan 45^\circ} = 30 \text{ m}$$

and in right triangle BTO

$$\tan 60^\circ = \frac{OT}{OB} = \frac{30}{y}$$

$$\Rightarrow y = \frac{30}{\tan 60^\circ} = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 17.32 \text{ m}$$

Hence, the required distance = x + y = 30 + 17.32 = 47.32 m

SOLVED PROBLEMS ON HEIGHTS AND DISTANCE:

1. 'a' and 'b' are the lengths of the base and height of a right angled triangle whose hypotenuse is 'h'. If the values of 'a' and 'b' are positive integers, which of the following cannot be a value of the square of the hypotenuse?

- (1) 13 (2) 23 (3) 37 (4) 41

Correct choice (2). Correct Answer - 23

1.Solution:

The value of the square of the hypotenuse = $h^2 = a^2 + b^2$

As the problem states that 'a' and 'b' are positive integers, the values of a^2 and b^2 will have to be perfect squares. Hence we need to find out that value amongst the four answer choices which cannot be expressed as the sum of two perfect squares.

Choice 1 is 13. $13 = 9 + 4 = 3^2 + 2^2$. Therefore, Choice 1 is not the answer as it is a possible value of h^2

Choice 2 is 23. 23 cannot be expressed as the sum two numbers, each of which in turn happen to be perfect squares. Therefore, Choice 2 is the answer.

Reasoning and Quantitative aptitude Heights and Distance

2. A spherical ball of radius 'r' placed on the ground subtends an angle of 60° at point A of the ground. What is the distance between the center of the ball and the point A?

- (1) r (2) 2r (3) 0.5r (4) none of these

Correct choice (2). Correct Answer - (2r)

Solution:

In an equilateral triangle all three sides are of the same length and let this be 'a' units.

From the diagram it is clear that OA is the angle bisector of angle LAM.

Therefore, angle OAL = 30°

In the right triangle OAL, $\sin 30^\circ = \frac{OL}{OA}$

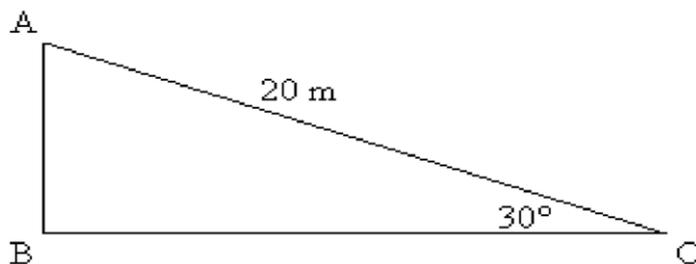
We know that OL is the radius of the sphere = r

Therefore, $\frac{1}{2} = \frac{r}{OA}$

Or OA = 2r

3. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

A. 8m B. 10m C. 12m D. 11m



Solution: In ΔABC $\sin 30^\circ = \frac{AB}{AC}$

Or, $\frac{1}{2} = \frac{AB}{20}$

Or, AB = 10 m

Reasoning and Quantitative aptitude Heights and Distance

4. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

A. $8\sqrt{3}$ B. $6\sqrt{3}$ C. 6 D. 8

Solution: Let us use the previous figure as a reference.

Here, $BC = 8$ m

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\text{Or, } \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\text{Or, } AB = \frac{8}{\sqrt{3}} = 4.61 \text{ m}$$

$$\text{Now, } \cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{BC}{AC} = \frac{8}{AC}$$

$$\text{Or, } AC = \frac{16}{\sqrt{3}}$$

$$\text{Height of tree} = AB + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

5. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

A. 2.598, $1/\sqrt{3}$ B. 2.598, $\sqrt{3}$ C. 1.5, $2\sqrt{3}$ D. 1.5, $4\sqrt{3}$

Solution: In the case of first slide

$$\tan 30^\circ = \frac{p}{b}$$

$$\text{Or, } \frac{1}{\sqrt{3}} = \frac{1.5}{b}$$

$$\text{Or, } b = \frac{3\sqrt{3}}{2} = 2.598 \text{ m}$$

In the case of second slide

$$\tan 60^\circ = \frac{p}{b}$$

$$\text{Or, } \sqrt{3} = \frac{3}{b}$$

$$\text{Or, } b = \frac{3}{\sqrt{3}} = 1.732 \text{ m}$$

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6. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30° . Find the height of the tower.

A.30m B.1.732m C.10m D.17.32m

Solution: $\tan 30^\circ = \frac{p}{30}$

Or, $p = \frac{30}{\sqrt{3}} = 10\sqrt{3} = 17.32 \text{ m}$

7. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60° . Find the length of the string, assuming that there is no slack in the string. A. $40\sqrt{3}$ B. $20\sqrt{3}$ C.60 D.40

Solution: $\sin 60^\circ = \frac{p}{h} = \frac{60}{h}$

Or, $\frac{\sqrt{3}}{2} = \frac{60}{h}$

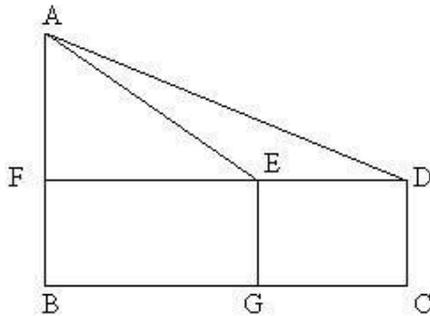
Or, $h = 60 \times \frac{2}{\sqrt{3}} = 40\sqrt{3}$

8. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

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A. $28.5\sqrt{3}$ B. $9.5\sqrt{3}$ C. $19\sqrt{3}$ D. $\sqrt{3}$



Solution: AB = 30 m (height of building)

DC = EG = 1.5 m (height of boy)

Angle D = 30°

Angle E = 60°

AF = 30 - 1.5 = 28.5 m

$$\text{In } \triangle AFD \tan 30^\circ = \frac{AF}{FD} = \frac{28.5}{FD}$$

$$\text{Or, } \frac{1}{\sqrt{3}} = \frac{28.5}{FD}$$

$$\text{Or, } FD = 28.5\sqrt{3}$$

$$\text{In } \triangle AFE \tan 60^\circ = \frac{AF}{FE} = \frac{28.5}{FE}$$

$$\text{Or, } \sqrt{3} = \frac{28.5}{FE}$$

$$\text{Or, } FE = \frac{28.5}{\sqrt{3}}$$

Required Distance = ED

$$= 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

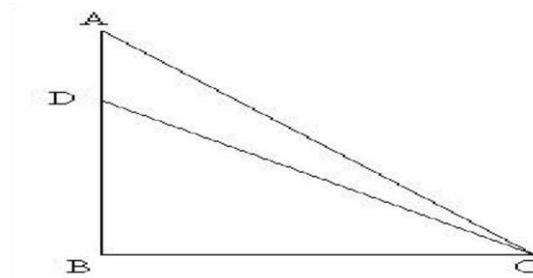
$$= 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$$

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9. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

A. 20.64m B. 14.64m C. 20m D. $\sqrt{3}$ m

Reasoning and Quantitative aptitude Heights and Distance



Solution: Height of Building = $DB = 20$ m

Angle $DCB = 45^\circ$

Angle $ACB = 60^\circ$

$$\text{In } \triangle DBC \tan 45^\circ = 1 = \frac{DB}{BC}$$

Or, $DB = BC = 20$ m (i)

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{AB}{BC}$$

$$\text{Or, } AB = 20\sqrt{3}$$

$$\text{Now, } AD = AB - DB$$

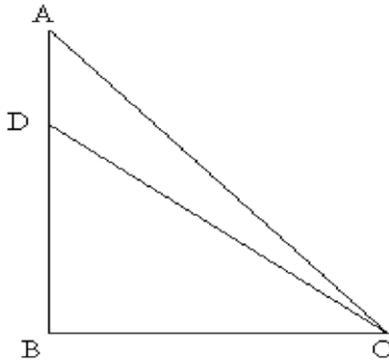
$$= 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$$

$$= 20(1.732 - 1) = 20 \times 0.732 = 14.64 \text{ m}$$

10. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.

- A. 1.6m B. $\sqrt{3}$ m C. 2.18m D. 2.6m

Reasoning and Quantitative aptitude Heights and Distance



Solution: Height of Statute = AD = 1.6 m

Angle ACB = 60°

Angle DCB = 45°

$$\text{In } \triangle DBC \tan 45^\circ = 1 = \frac{DB}{BC}$$

Or, BC = DB

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{AB}{BC}$$

$$\text{Or, } BC = \frac{AB}{\sqrt{3}} = \frac{BC + 1.6}{\sqrt{3}} \quad (\text{as } AB = DB + AD = BC + AD)$$

$$\text{Or, } BC\sqrt{3} - BC = 1.6$$

$$\text{Or, } BC(\sqrt{3} - 1) = BC(1.732 - 1) = 1.6$$

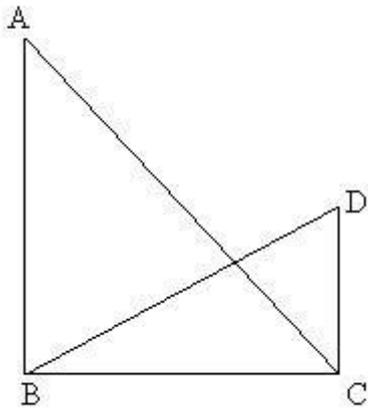
$$\text{Or, } BC = \frac{1.6}{0.732} = 2.18 \text{ m}$$

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11. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

- A. $50/3$ m B. 50m C. $50/\sqrt{3}$ D. 16m

Reasoning and Quantitative aptitude Heights and Distance



Solution: Height of Tower = $AB = 50$ m

Angle $ACB = 60^\circ$

Angle $DBC = 30^\circ$

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{50}{BC}$$

$$\text{Or, } BC = \frac{50}{\sqrt{3}}$$

$$\text{In } \triangle DCB \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{DC}{BC}$$

$$\text{Or, } DC = \frac{50}{3} = 16\frac{2}{3} \text{ m}$$

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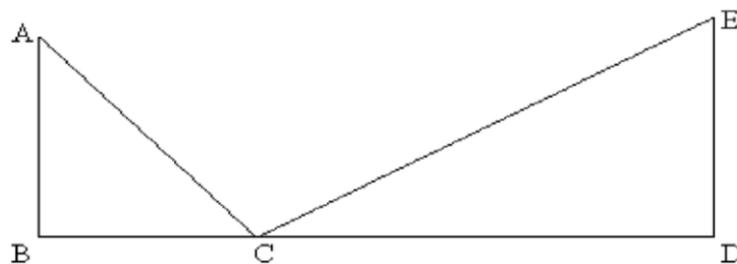
12. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles.

A. $20\sqrt{3}$ m, 20m

B. 28m, $20\sqrt{3}$ m

C. 80m, 20m

D. 20m, 80m



Reasoning and Quantitative aptitude Heights and Distance

Solution: BD = Width of Road = 80 m

Angle ACB = 60°

Angle ECD = 30°

AB = ED = Height of poles

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{AB}{BC}$$

$$\text{Or, } AB = BC\sqrt{3} \dots\dots\dots(i)$$

$$\text{In } \triangle EDC \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{ED}{CD} = \frac{AB}{80 - BC}$$

$$\text{Or, } AB = (80 - BC) \times \frac{1}{\sqrt{3}} \dots\dots\dots(ii)$$

From equation(i) and (ii)

$$BC\sqrt{3} = (80 - BC) \times \frac{1}{\sqrt{3}}$$

$$\text{Or, } 3BC = 80 - BC$$

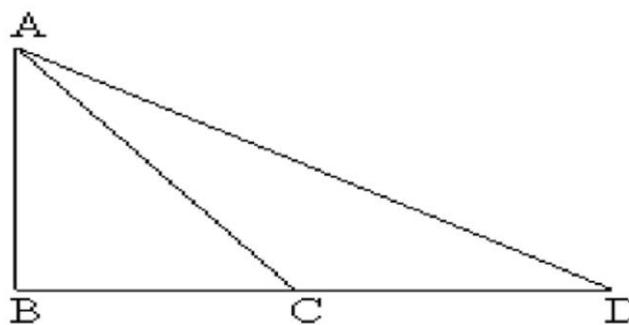
$$\text{Or, } 4BC = 80$$

$$\text{Or, } BC = 20$$

$$\text{So, } AB = 20\sqrt{3} \text{ m}$$

13. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

- A.10m,20m B.30m,20m C.10 $\sqrt{3}$ m,10m D.20m,10 $\sqrt{3}$ m



Reasoning and Quantitative aptitude Heights and Distance

Solution: Width of canal = BC

CD = 20 m

Angle ACB = 60°

Angle ADB = 30°

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{AB}{BC}$$

$$\text{Or, } AB = BC\sqrt{3}$$

$$\text{In } \triangle ABD \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$$

$$\text{Or, } AB = \frac{BC + 20}{\sqrt{3}}$$

$$\text{Or, } BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$$

$$\text{Or, } 3BC = BC + 20$$

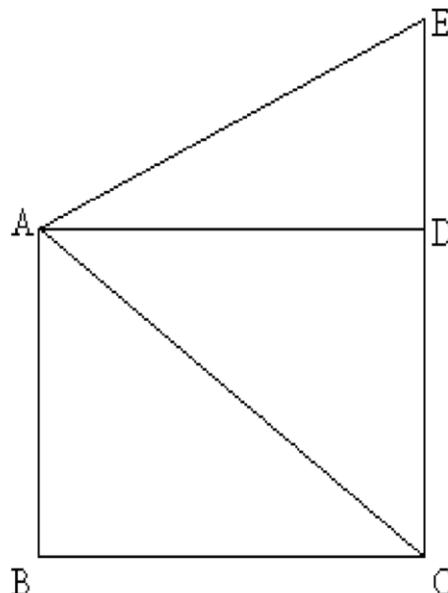
$$\text{Or, } BC = 10 \text{ Width of canal}$$

$$\text{Height of tower } AB = 10\sqrt{3}$$

14. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.

A. 19.124m B. $7\sqrt{3}$ m C. 7m D. 191.24m

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Reasoning and Quantitative aptitude Heights and Distance

Solution: AB = Height of Building = 7 m

Angles ACB = 45°

Angle EAD = 60°

$$\text{In } \triangle ABC \tan 45^\circ = 1 = \frac{AB}{BC}$$

$$\text{Or, } AB = BC = AD = 7 \text{ m}$$

$$\text{In } \triangle EAD \tan 60^\circ = \sqrt{3} = \frac{ED}{AD}$$

$$\text{Or, } ED = 7\sqrt{3}$$

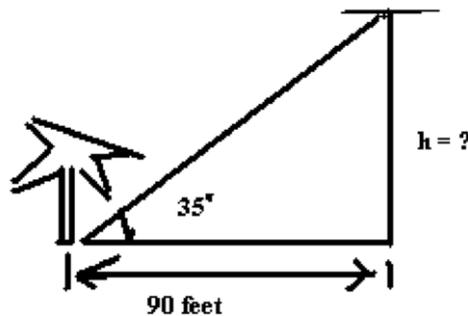
$$\text{So height of tower} = DC + ED = 7 + 7\sqrt{3}$$

$$= 7(1 + \sqrt{3}) = 19.124 \text{ m}$$

15. A girl is sitting in the shade under a tree that is 90 ft from the base of a tower. The angle of elevation from the girl to the top of the tower is 35 degrees. Find the height of the windmill.(in feet)

A. 40 B. 64 C. 42.64 D. 40.62

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Solution:

Here given the the girl is 90 feet from the tower

The angle of elevation from the girl to the tower is 35°

Here we want to solve and find the height of the tower

Recall the trigonometry formulas

Here the angle and the adjacent side length is given

So use the formula of tan tan

Reasoning and Quantitative aptitude Heights and Distance

$$\begin{aligned} 35^\circ &= \text{opposite} / \text{adjacent} \\ \tan 35^\circ &= h / 90 \quad h = 90 * \tan \end{aligned}$$

$$35^\circ h = 90 * 0.4738 \quad h =$$

$$42.64 \text{ feet}$$

Thus the height of the tower is 42.64 feet.

EXERCISE PROBLEMS WITH SOLUTIONS:

1. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

A.75m B. $75\sqrt{3}$ m C.50m D.54.9m

2. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.

A. $58\sqrt{3}$ B. $87/\sqrt{3}$ C. $87\sqrt{3}$ D. $58/\sqrt{3}$

3. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30° , which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower from this point.

A. $AB/\sqrt{3}$ B. $AB\sqrt{3}$ C. $A/\sqrt{3}$ D. $B\sqrt{3}$

4. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

A.4m B.6m C.9m D.7m

Reasoning and Quantitative aptitude Heights and Distance

5. The angle of depression of a vehicle on the ground from the top of a tower is 60° . If the vehicle is at a distance of 100 meters away from the building, find the height of the tower.

A. 173.20m B. 175m C. 100m D. 170m

6. The angle of depression of a stone on the ground from the top of a tower is 45° . If the stone is at a distance of 120 meters away from the building, find the height of the tower.

A. 100m B. 150m C. 120m D. 110m

7. From a cliff 150m above the shore line the angle of depression of a ship is $19^\circ 30'$.

Find the distance from the ship to a point on the shore directly below the observer.

A. 150m. B. 420m C. 425m D. 423.59 m

8. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60° . When he was 40m away from the bank he finds that the angle of elevation to be 30° . Find the (i) height of the tree, (ii) The width of the river, correct to two decimal places.

A. 40m, 20m B. 34m, 64m C. 34.64m, 20m D. 20m, 64m

9. The angle of elevation of the top of a tower 30 m high, from two points on the level ground on its opposite sides are 45 degrees and 60 degrees. What is the distance between the two points?

A. 30 B. 51.96 C. 47.32 D. 81.96

10. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

Reasoning and Quantitative aptitude Heights and Distance

A.10m,20m B. $10\sqrt{3}$ m, 10m C. 30m, 20m D.20m, $10\sqrt{3}$ m **ANSWERS AND SOLUTIONS:**

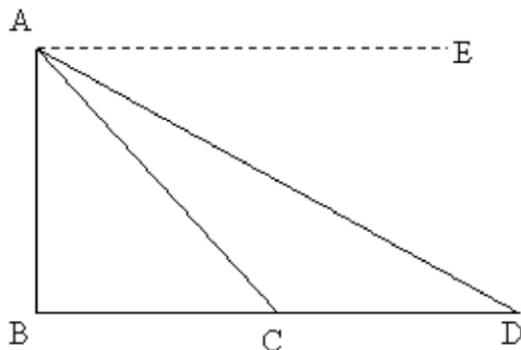
ANSWERS:

7. D 8. C 9.C 10.B

1. D 2. A 3. A 4. B 5. A 6. C

SOLUTIONS:

1.



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Solution: AB = Height of Lighthouse = 75 m

Angle EAC = Angle ACB = 45°

Angle EAD = Angle ADB = 30°

$$\text{In } \triangle ABC \tan 45^\circ = 1 = \frac{AB}{AC}$$

Or, AB = AC = 75 m

$$\text{In } \triangle ABD \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

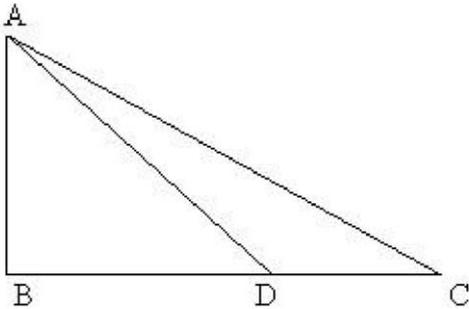
Or, $BD = 75\sqrt{3}$

So, Distance between ships C and D

$$CD = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1) = 54.9 \text{ m}$$

Reasoning and Quantitative aptitude Heights and Distance

3.



Solution: AB = Height of tower

Angle ADB = 60°

Angle ACB = 30°

$$\text{In } \triangle ABD \tan 60^\circ = \sqrt{3} = \frac{AB}{BD}$$

$$\text{Or, } BD = \frac{AB}{\sqrt{3}}$$

$$\text{In } \triangle ABC \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

$$\text{Or, } BC = BD + DC = AB\sqrt{3}$$

$$DC = BC - BD$$

$$\text{Or, } DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$$

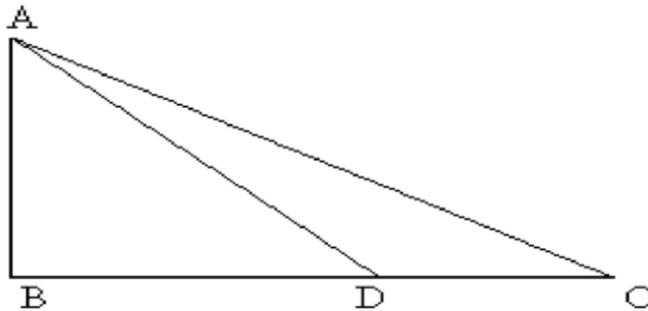
\therefore To travel $\frac{2AB}{\sqrt{3}}$ it takes 6 seconds

\therefore to travel $\frac{AB}{\sqrt{3}}$ it will take 3 seconds

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Reasoning and Quantitative aptitude Heights and Distance

4.



Solution: $BD = 4$ m

$BC = 9$ m

Angle $ADB = \theta$

Angle $ACB = 90^\circ - \theta$

In $\triangle ABD$ $\tan \theta = \frac{AB}{BD}$

Or, $AB = \tan \theta \times BD = \tan \theta \times 4$

In $\triangle ABC$ $\tan(90^\circ - \theta) = \frac{AB}{BC}$

Or, $AB = \cot \theta \times 9$

Or, $4 \tan \theta = 9 \cot \theta$

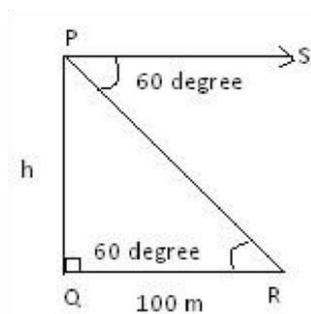
Or, $\tan^2 \theta = \frac{9}{4}$

Or, $\tan \theta = \frac{3}{2} = \frac{AB}{4}$

Or, $AB = 6$ m

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5.



R is a vehicle.

PQ is the height of the tower.

Reasoning and Quantitative aptitude Heights and Distance

RQ is the distance between the tower and the vehicle.

PS is the line of sight.

Angle of depression $\angle SPR = 60$ degree.

Angle of elevation $\angle QRP =$ Angle of depression $\angle SPR = 60$ degree.

So, it forms right angle triangle.

In right angled triangle PQ, we know that, $QR = 100$ m , angle $\theta = 60^\circ$, $PQ = h$ m

In trigonometry, we know that,

$$\text{Tan } \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{Tan } 60^\circ = \frac{PQ}{RQ} = \frac{h}{100}.$$

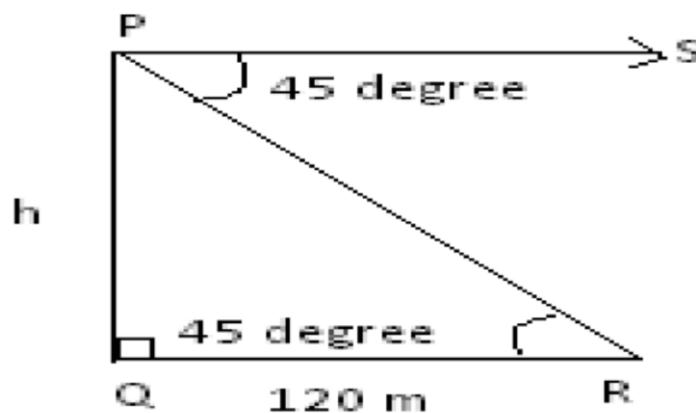
Apply cross multiplication,

$$h = 100 * \tan 60^\circ$$

173.20

The Height of the tower from ground is 173.20 meter.

6.



The above diagram is shown for question. In that, R is a stone.

Reasoning and Quantitative aptitude Heights and Distance

PQ is the height of the tower.

RQ is the distance between the tower and the stone.

PS is the line of sight.

Angle of depression $\angle SPR = 45$ degree.

Angle of elevation $\angle QRP =$ Angle of depression $\angle SPR$

$= 45$ degree.

So, It forms right angle triangle.

In right angled triangle PQ, we know that, $QR = 120$ m , angle $\theta = 45^\circ$, $PQ = h$ m

In trigonometry, we know that,

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan 45^\circ = \frac{PQ}{RQ} = \frac{h}{120}$$

Apply cross multiplication,

$$h = 120 * \tan 45^\circ \quad h = 120$$

The Height of the tower is 120 meter.

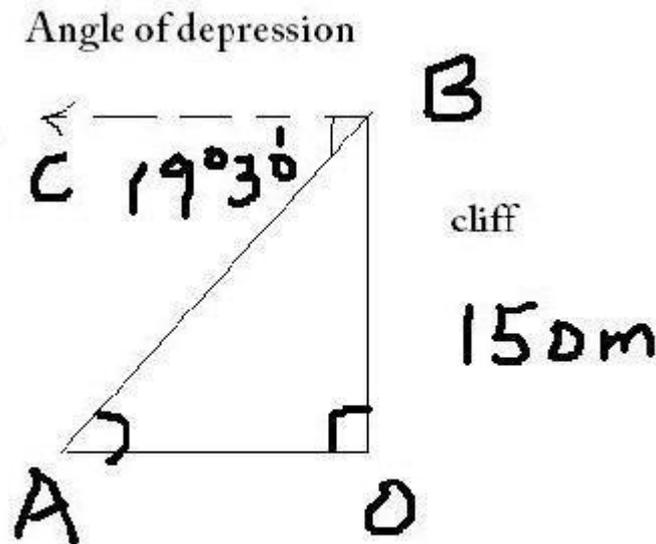
7.

Let OB be the cliff of height 150 m , A be the position of the ship the angle of depression of the ship is $19^\circ 30'$,

$$\angle ABC = 19^\circ 30' .$$

Reasoning and Quantitative aptitude Heights and Distance

$$\angle ABC = 19^{\circ} 30' \Rightarrow \angle OAB = \angle ABC = 19^{\circ} 30'$$



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In Right angled triangle ΔAOB , we want to find $OA =$ adjacent side,
 $OB =$ opposite side = 150m

$$\cot (19^{\circ} 30') = \frac{OA}{OB} = \frac{OA}{150}$$

$$OA = 150 \cot (19^{\circ} 30') = 150 \tan (90 - 19^{\circ} 30') \quad OA = 150 *$$

$$\tan (70^{\circ} 30') = 150 (2.8239) = 423.59 \text{ m}$$

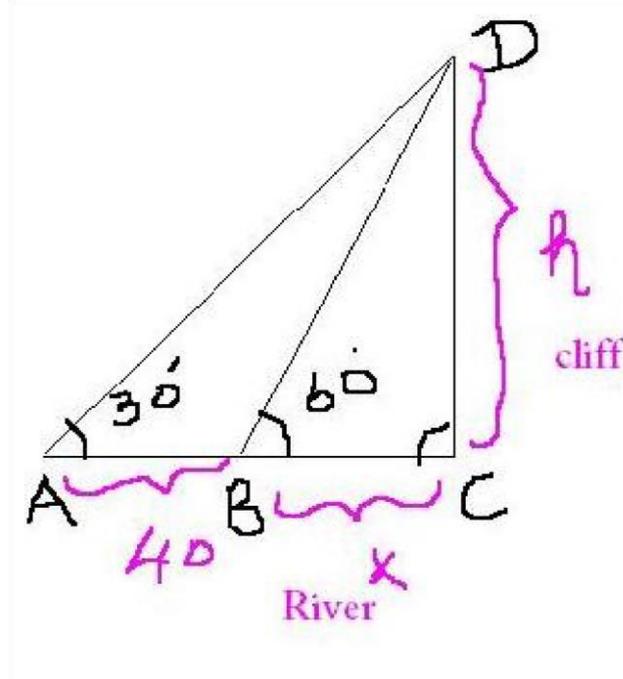
8.

Let CD is the tree , $CD = h$ m high

BC is the river , $BC = x$ width of the river

Reasoning and Quantitative aptitude Heights and Distance

The observer is at A , From figure $\angle DAC = 30^\circ$ and $\angle DBC = 60^\circ$



In right angled $\triangle BCD$, x is the adjacent side , h is the opposite side .

$$\tan 60^\circ = \frac{DC}{BC} \Rightarrow \sqrt{3} = \frac{DC}{BC} =$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow \sqrt{3}x = h \text{ -----(1)}$$

From right angled $\triangle ACD$, $\tan 30^\circ = \frac{DC}{AC} \quad AC = 40 + x$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+40} \Rightarrow x+40 = \sqrt{3}h \text{ -----(2)}$$

Plugging in value of h from 1 in 2

$$x + 40 = \sqrt{3} (x + \sqrt{3})$$

$$x + 40 = 3x$$

$$40 = 2x \quad \text{divide by 2 , } x = 20$$

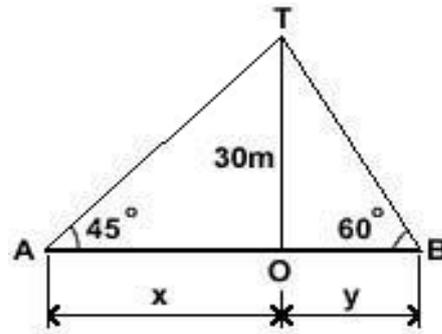
Reasoning and Quantitative aptitude Heights and Distance

$$h = \sqrt{3} x = 1.732 (20) = 34.64 [\sqrt{3} = 1.732]$$

The height of the tree = 34.64 m

Width of the river = 20 m

9.



Let OT be the tower. Therefore,

Height of tower = OT = 30 m

Let A and B be the two points on the level ground on the opposite side of tower OT.

Then, angle of elevation from A = $\angle TAO = 45^\circ$ and angle of elevation from B = $\angle TBO = 60^\circ$

Distance between AB = AO + OB = x + y (say)

Now, in right triangle ATO,

$$\tan 45^\circ = \frac{OT}{AO} = \frac{30}{x}$$

$$\Rightarrow x = \frac{30}{\tan 45^\circ} = 30 \text{ m}$$

and in right triangle BTO

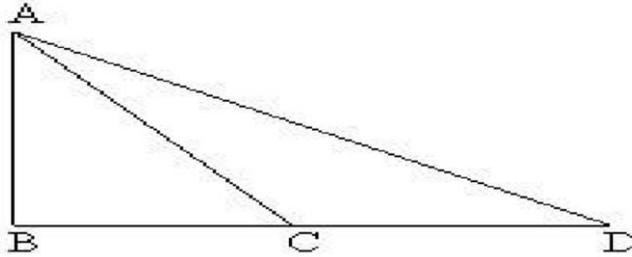
$$\tan 60^\circ = \frac{OT}{OB} = \frac{30}{y}$$

$$\Rightarrow y = \frac{30}{\tan 60^\circ} = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 17.32 \text{ m}$$

Hence, the required distance = x + y = 30 + 17.32 = 47.32 m

Reasoning and Quantitative aptitude Heights and Distance

10.



Solution: Width of canal = BC

CD = 20 m

Angle ACB = 60°

Angle ADB = 30°

$$\text{In } \triangle ABC \tan 60^\circ = \sqrt{3} = \frac{AB}{BC}$$

$$\text{Or, } AB = BC\sqrt{3}$$

$$\text{In } \triangle ABD \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$$

$$\text{Or, } AB = \frac{BC + 20}{\sqrt{3}}$$

$$\text{Or, } BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$$

$$\text{Or, } 3BC = BC + 20$$

$$\text{Or, } BC = 10 \text{ Width of canal}$$

$$\text{Height of tower } AB = 10\sqrt{3}$$

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